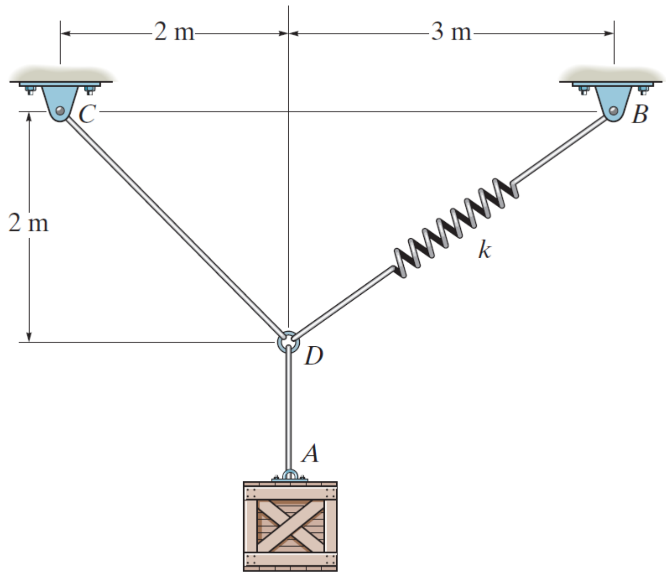
Problem 3-19

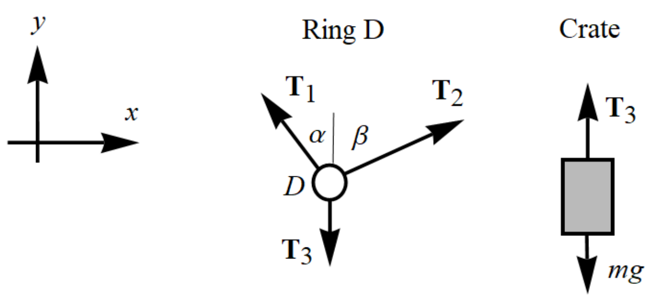
Determine the unstretched length of DB to hold the 40-kg crate in the position shown. Take $k = 180$ N/m.



Probs. 3–18/19

Solution

Draw one free-body diagram for the ring at D and one free-body diagram for the crate.



In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad T_2 \sin\beta - T_1 \sin\alpha = 0 \qquad 0 = 0$$

$$\sum F_y = 0: \qquad T_1 \cos\alpha + T_2 \cos\beta - T_3 = 0 \qquad T_3 - mg = 0$$

Since $T_3 = mg$ and $T_2 = k\Delta x_{DB} = (180 \text{ N/m})(\sqrt{2^2 + 3^2} \text{ m} - x_0)$, the system of equations reduces to

$$180\left(\sqrt{2^2+3^2} - x_0\right)\sin\beta - T_1 \sin\alpha = 0 \tag{1}$$

$$T_1 \cos\alpha + 180\left(\sqrt{2^2+3^2} - x_0\right)\cos\beta - mg = 0. \tag{2}$$

Solve equation (1) for T_1

$$T_1 = \frac{180\left(\sqrt{2^2+3^2} - x_0\right)\sin\beta}{\sin\alpha}$$

and substitute it into equation (2). Solve for x_0.

$$\frac{180\left(\sqrt{2^2+3^2} - x_0\right)\sin\beta}{\sin\alpha}\cos\alpha + 180\left(\sqrt{2^2+3^2} - x_0\right)\cos\beta - mg = 0$$

$$180\left(\sqrt{2^2+3^2} - x_0\right)\sin\beta\cot\alpha + 180\left(\sqrt{2^2+3^2} - x_0\right)\cos\beta = mg$$

$$\left(\sqrt{2^2+3^2} - x_0\right) = \frac{mg}{180\sin\beta\cot\alpha + 180\cos\beta}$$

$$x_0 = \sqrt{2^2+3^2} - \frac{mg}{180\sin\beta\cot\alpha + 180\cos\beta}$$

Use trigonometry to determine α and β.

$$\tan\alpha = \frac{2}{2} \quad\rightarrow\quad \alpha = \tan^{-1}(1) = 45^\circ$$

$$\tan\beta = \frac{3}{2} \quad\rightarrow\quad \beta = \tan^{-1}\left(\frac{3}{2}\right) \approx 56.3^\circ$$

Therefore, since $m = 40$ kg and $g = 9.81$ m/s^2, the spring's unstretched length is

$$x_0 \approx 2.03 \text{ m}.$$